Journal of Sound and Vibration (1997) **200**(2), 235–239

AN IMPROVED MODEL FOR PREDICTING FUNDAMENTAL FREQUENCIES OF PLATES CARRYING MULTIPLE MASSES

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(*Received* 20 *October* 1995, *and in final form* 28 *May* 1996)

1. INTRODUCTION

In a previous paper [1], an equivalent center-weight factor (ECWF) method was used to predict the fundamental frequencies of the loaded plates carrying multiple masses by using merely the data measured earlier for the respective plates with a single mass. It was pointed out that an alternative form of Dunkerley's formula and the ECWF method enable one to obtain analytically a quick and relatively accurate estimation of the fundamental frequencies of plates carrying concentrated masses. In this letter an improved model for a quick and accurate estimation of the fundamental frequency is presented by taking into account the change in the strain energy owing to a different loading condition on the same plate.

2. FREQUENCY ESTIMATION FOR PLATES WITH MULTIPLE MASSES

The equating of the strain energy to the kinetic energy yields the final expression of Rayleigh's quotient [2, 3]

$$\omega^2 = U_{max}/T^* \tag{1}$$

where $T^* = T_{max}/\omega^2$ is called reference kinetic energy (a list of nomenclature is given in the Appendix). Its analogous expression for a vibrating plate carrying a concentrated mass is [4]

$$\omega^2 = \frac{\displaystyle\int_0^b \int_0^a D\left\{\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1-\nu)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right]\right\} \mathrm{d}x\,\mathrm{d}y}{\displaystyle\int_0^b \int_0^a \gamma(x, y) w^2(x, y)\,\mathrm{d}x\,\mathrm{d}y + M(\zeta, \eta) w^2(\zeta, \eta)}, \tag{2}$$

in which the shape function $w(\zeta, \eta)$ is associated with the concentrated mass $M(\zeta, \eta)$ at the plate's co-ordinates (ζ, η). In the case of the plate carrying finite masses, equation (2) can be extended to introduce the respective concentrated masses $M_j(\zeta_j, \eta_j)$ located at co-ordinates (ζ_j, η_j).

The following symbolic form has been introduced for equation (2) [1]:

$$\omega^2 = k^*/(m^* + M^*), \tag{3}$$

where M^* is the generalized term associated with the concentrated mass M, while m^* is the generalized quantity associated with the plate only, and k^* is the generalized stiffness associated with the complete system.

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In view of equations (2) and (3), the fundamental frequency for a plate carrying a concentrated mass (M_1) at the plate's centre can be expressed as

$$\omega_1^2 = \frac{\int_0^b \int_0^a D\left\{\left(\frac{\partial^2 w}{\partial x^2} + \frac{\partial^2 w}{\partial y^2}\right)^2 - 2(1-v)\left[\frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial y^2} - \left(\frac{\partial^2 w}{\partial x \partial y}\right)^2\right]\right\} \mathrm{d}x\,\mathrm{d}y}{\int_0^b \int_0^a \gamma(x, y) w^2(x, y)\,\mathrm{d}x\,\mathrm{d}y + M_1(a/2, b/2) w^2(a/2, b/2)} = \frac{k^*}{m^* + M_1^*}. \quad (4)$$

On the other hand, an expression can be obtained if the mass at the plate's centre is treated as a combination of the two mass components, $M_1(\zeta_1, \eta_1)$ and $M_2(\zeta_2, \eta_2)$, placed at locations (ζ_1, η_1) and (ζ_2, η_2) respectively:

$$\omega^2 = k^*/(m^* + M_1^* + M_2^*). \quad (5)$$

To predict the system frequency (ω) of a plate carrying multiple masses, merely from the individual frequency (ω_j) and the unloaded frequency (ω_0), an alternative form of Dunkerley's formula was derived [1]:

$$\frac{1}{\omega^2} = \sum_j \left(\frac{1}{\omega_j^2}\right) - (j-1)\frac{1}{\omega_0^2}, \quad (6)$$

where $\omega_0^2 = k^*/m^*$ and j denotes the number of concentrated masses on the plate surface.

The quantity k^* of equation (3) was assumed to be constant in reference [1] for the frequencies, ω_1 and ω_2, of plates carrying different masses. It is an approximation which implies that the change in strain energy (U) owing to the weight difference is not significant. To account for the change in the strain energy term, equation (6) should be replaced by

$$\frac{1}{\omega^2} = \frac{k_0^*}{k^*}\sum_j \left(\frac{k_j^*/k_0^*}{\omega_j^2}\right) - (j-1)\frac{1}{\omega_0^2}, \quad (7)$$

where

$$\omega_j^2 = k_j^*/(m^* + M_j^*). \quad (8)$$

3. COMPARISON OF RESULTS

To verify the validity of the proposed model, a vibration test was conducted for a 600×300 mm plate carrying different masses at its centre (see reference [1] for details of the experimental set-up). The results are summarized in Table 1, where f_1 is the fundamental frequency of the plate obtained experimentally. It is interesting to note from Figure 1 and Table 1 that the ratio k^*/k_0^* decreases sharply for $0 < M_c/m < 0{\cdot}255$ but decreases gradually for $M_c/m > 0{\cdot}255$. As mentioned in the previous section, the fundamental frequency of a complete system can be extracted from those frequencies associated with individual mass components. As shown in Table 2, the system frequency obtained experimentally has been compared with that estimated by using equation (6) or equation (7). The value of k^*/k_0^* with respect to each different mass ratio is found from

TABLE 1

Result for the 600 × 300 mm plate carrying a concentrated mass

Mass number	M_c (g)	M_c/m	f_1 (Hz)	k^* (kN/m²)	k^*/k_0^*
—	**0·00**	**0·000**	**145·80**	**995·31**	**1·000**
M1	10·33	0·009	141·50	945·64	0·950
M2	20·48	0·017	137·80	904·44	0·909
M3	30·42	0·026	134·30	866·15	0·870
M4	40·55	0·034	130·80	828·44	0·832
M5	50·30	0·042	127·50	793·42	0·797
M6	60·44	0·051	124·50	762·73	0·766
M7	70·65	0·060	121·50	732·36	0·736
M8	80·75	0·068	118·80	705·80	0·709
M9	90·92	0·077	116·30	681·84	0·685
M10	101·10	0·085	114·30	663·59	0·667
M11	161·87	0·140	104·50	576·33	0·579
M12	202·20	0·170	97·75	522·84	0·525
M13	303·00	0·255	87·50	449·35	0·451
M14	403·90	0·341	84·00	441·96	0·444
M15	506·70	0·427	81·25	439·66	0·442
M16	607·80	0·512	79·25	443·23	0·445
M17	708·90	0·598	76·75	438·85	0·441
M18	809·80	0·683	74·50	435·54	0·438
M19	910·80	0·768	72·50	433·17	0·435
M20	1005·5	0·848	71·00	435·22	0·437
M21	1106·7	0·933	69·50	436·30	0·438
M22	1207·8	1·018	68·00	435·78	0·438
M23	1308·9	1·104	66·50	434·33	0·436
M24	1411·5	1·190	65·00	431·59	0·434
M25	1512·2	1·275	63·75	431·25	0·433

$m = 1{\cdot}186$ kg.

Figure 1. It is obvious that by accounting for the change of the ratio k^*/k_0^*, equation (7) represents a better model that can well predict the frequency of the complete system, especially for systems involving heavy masses.

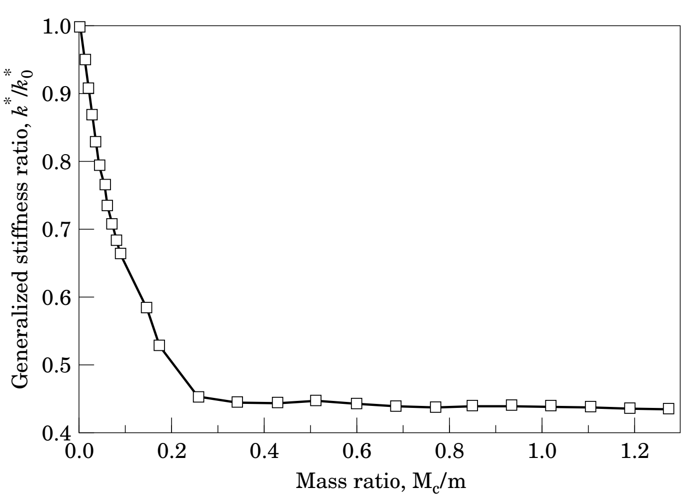

Figure 1. The curve for k^*/k_0^* versus M_c/m.

TABLE 2

Estimation of the system frequency for the 600 × 300 *mm plate carrying concentrated masses from Table* 1

Mass combination	f_{est1} (Hz), equation (6)	f_{est2} (Hz), equation (7)	f_{exp} (Hz), experiment	Error 1 (%)	Error 2 (%)
M1 + M2	134·15	134·28	134·30	−0·11	−0·02
M1 + M10	119·41	118·79	118·80	0·51	−0·01
M6 + M9	104·60	104·48	104·50	0·09	−0·02
M1 + M2 + M3	125·03	124·46	124·50	0·43	−0·03
M1 + M5 + M10	104·91	104·47	104·50	0·39	−0·02
M4 + M6 + M8	97·39	97·69	97·75	−0·36	−0·06
M1 + M2 + M3 + M4	115·18	114·25	114·30	0·77	−0·05
M2 + M4 + M5 + M6	105·24	104·47	104·50	0·71	−0·02
M1 + M3 + M5 + M9	97·26	97·67	97·75	−0·50	−0·08
M2 + M4 + M6 + M8	85·90	87·45	87·50	−1·83	−0·06
M1 + M2 + M3 + M4 + M5	105·50	104·45	104·50	0·96	−0·05
M2 + M3 + M4 + M5 + M6	97·86	97·67	97·75	0·11	−0·08
M1 + M2 + M4 + M6 + M7	86·22	87·44	87·50	−1·46	−0·07
M10 + M14	66·62	76·75	76·75	−13·19	0·00
M13 + M14	67·91	74·50	74·50	−8·85	0·00
M12 + M17	66·31	72·50	72·50	−8·53	0·01
M15 + M19	58·25	65·00	65·00	−10·38	−0·02
M17 + M18	57·46	63·75	63·75	−9·87	−0·02
M10 + M12 + M13	67·77	79·25	79·25	−14·48	0·01
M10 + M16 + M17	56·66	65·00	65·00	−12·82	−0·04
M10 + M15 + M19	54·30	63·75	63·75	−14·84	−0·03
M10 + M12 + M13 + M14	56·58	70·98	71·00	−20·31	−0·02
M10 + M12 + M15 + M16	53·39	64·98	65·00	−17·86	−0·03
M10 + M12 + M15 + M17	52·61	63·74	63·75	−17·48	−0·02

Note: error $\# = (f_{est\,\#} - f_{exp})/f_{exp}$.

4. CONCLUDING REMARKS

This work presents an improved model to estimate the fundamental frequency of plates carrying concentrated masses from those frequencies associated with individual mass components. It is found that the change in the strain energy should be incorporated in the model, especially for cases of large masses, in order to predict well the natural frequency of the complete system from those of the component systems. The study is useful in the frequency estimation, either calculated or determined experimentally. The work can also be extended to off-centre loading systems by making use of the equivalent center-weight method [1].

It should be noted that the model in reference [1] was based on Dunkerley's method, where an additional mass is attached to an existing system. Complementary to Dunkerley's method is Southwell's method, which is particularly useful where some additional stiffness is attached or constraint made to an existing system. Both of the methods can be derived from the Rayleigh quotient [5]. In fact, the improved model presented here is originated from the expression of Rayleigh's quotient, equation (1).

ACKNOWLEDGMENTS

The authors are indebted to Hung-Heng Tan for his great assistance in conducting the experimental analysis. The authors would also like to extend their gratitude to the reviewer for his/her constructive comments.

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APPENDIX: NOMENCLATURE

a, b	dimensions of plate
D	$=Eh^3/12(1-v^2)$, flexural rigidity of plate
E	Young's modulus of plate material
f	fundamental frequency of the loaded plate (Hz)
f_0	fundamental frequency of the unloaded plate (Hz)
h	plate thickness
j	number of concentrated masses on the plate surface
k^*	generalized stiffness associated with the complete system
k_j^*	generalized stiffness associated with the loaded plate carrying a mass M_j
k_0^*	generalized stiffness associated with the unloaded plate
m^*	generalized mass associated with the plate only
m	mass of the unloaded plate
M	concentrated mass mounted on the plate surface
M^*	generalized mass associated with the concentrated mass M
T	kinetic energy
U	strain energy
w	transverse plate displacement function
γ	density of plate (kg/m^2)
ω	$=2\pi f$, fundamental frequency of the loaded plate (rad/sec)
ω_0	$=2\pi f_0$, fundamental frequency of the unloaded plate (rad/sec)
v	Poisson ratio of plate material
ζ, η	co-ordinates of weight placed on the plate surface